Reply to the comment by J W Lee. Self-attracting walk: are the exponents universal?

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COMMENT

Reply to the comment by J W Lee. Self-attracting walk: are the exponents universal?

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Abstract. Problems concerning the value of the exponent ν in the relationship $\xi^2 \sim t^{2\nu}$ describing a self-attracting walk are considered. It is suggested that further study is needed to understand how the exponent depends on the coupling energy of the walk.

I welcome the comment by Lee aimed at further study of the self-attracting walk model [1]. At the same time, I would like to clarify several aspects of my paper discussed in the comment. In the paper, a model of a self-attracting walk (SATW) was proposed and the following theoretical relationship was derived for the model

$$\nu = 1/(2D - D_\mathrm{b}) \tag{1}$$

which connected the fractal exponent ν in the $\xi^2 \sim t^{2\nu}$ relation with the fractal dimension D of the cluster of visited sites and the fractal dimension D_b of the boundary of this cluster.

Using this equation, a value $\nu = 1/2$ was obtained for a one-dimensional walk [1, p L151]. This result follows immediately from equation (1) if one notices that in a one-dimensional case $D = 1$, as the cluster of visited sites is just an interval (it cannot have holes) and $D_\mathrm{b} = 0$ because the boundary consists of two endpoints of the interval. Thus, the computer simulation result $\nu = 1/2$ in one dimension obtained by Lee *confirms* my result rather than contradicting it. Previously this result was also confirmed by Prasad *et al* [2].

In two dimensions, the cluster of visited sites was found to be compact ($D = 2$) at least for $-u > 1.0$, and the values of D_b and ν were estimated by computer simulation. For a three-dimensional walk, based on relationship (1) it was concluded that *if the cluster of visited sites is compact, i.e., if* $D = 3$, then $1/4 < \nu < 1/3$. This follows from equation (1) if one takes into account the fact that in three dimensions $2 < D_\mathrm{b} < 3$. In the comment by Lee, the geometry of the cluster of visited sites was not studied. It is reasonable to suppose that at small values of $-u$ the cluster was not compact and thus $D < 3$ accounts for the $\nu > 1/3$ values obtained by Lee for $-u \leqslant 1.5$. More complicated are estimation issues (briefly discussed below) which are especially important for high values of $-u$ and which may have led to underestimating the ν values (e.g., the value of $\nu = 0.19$ given in the comment for $-u = 3$). Further study is needed to clarify this issue.

Except for the one-dimensional case, no claim was made in my paper that the exponent ν was universal (neither did I claim the opposite) because the data obtained did not permit a definite conclusion on that matter. The main problem consisted in $\xi^2(t)$ plots showing a long transitional behaviour before the asymptotic values were reached (the higher the $-u$ value the longer the transition). Clarification of this issue would be of considerable interest. It does not seem very clear if in Lee's work the number of jumps was high enough to permit one or the other conclusion. A similar problem applies to the hypothesis proposed in my paper that there exists a critical value u_c such that $\nu = 1/2$ if $0 < -u < -u_c$. To disprove this hypothesis one would have to verify that, for any value of u, the estimated difference between ν and 0.5 is significant and does not reflect transitional behaviour. This has not been done in the comment.

In the comment it is said that 'In Sapozhnikov's article there is an ambiguity regarding the scaling between bulk cluster and boundary cluster visited by the walk'. This claim is not elaborated enough to make clear what is really meant. However, it seems to correspond to the claim made in [3, p 3857] that for a SATW the probability of localizing the walking particle is not the same over the visited sites when $N \to \infty$. Thus, if it is implied in the comment that the walking particle visits boundary sites with different frequency than the other cluster sites, this claim, as well as the claim made in [3], cannot be correct for thermodynamic reasons. Indeed, in equilibrium (and the system evolves towards the equilibrium growth), the concentration of the walking particle can be different at the boundary sites only if this were justified by a different energy of the particle at the boundaries (the reader is reminded that the boundary consists of visited sites having non-visited neighbours). However, if this were the case, then the probability of the walking particle jumping into the boundary site would have been different from the probability of jumping into other visited sites. This contradicts the model where jump probabilities depend only on whether the sites have been visited before and not on whether the adjacent sites have been visited. For the same reason, for $N \to \infty$ (equilibrium cluster growth) the probability of being visited is the same for all previously visited sites.

References

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[3] Aarão Reis F D A 1995 *J. Phys. A: Math. Gen.* **28** 3851